THE UNIVERSITY OF ALBERTA

AN INVESTIGATION OF ABILITIES OF GRADE IV, V, AND VI PUPILS IN SOLVING AND ANALYZING PROBLEMS IN ARITHMETIC

A DISSERTATION

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CHAPTER I

PURPOSE OF THE INVESTIGATION

The schools are faced with the problem of taking in children who have little knowledge of arithmetic or number and an unknown and developing capacity for acquiring it, and turning them out so that they may be able to handle their own and others' affairs in an efficient way. How is it to be done?

Teaching methods and teaching instruments have until recently been entirely the result of teachers' experience. That which has seemed best has been accepted and changes were introduced very slowly. Since the advent of the experimental method in psychology there has been a much more scientific attempt to discover underlying laws and principles, and to improve methods of teaching and school practice. If for no other reason, this has become necessary due to the ever increasing body of material which must be learned by the child if he is to be prepared for life. Investigations in the field of arithmetic have aided teachers in determining the best methods to be used in teaching the fundamental operations, the amount of drill necessary, the order of presentation of number facts, and the amount of review necessary to keep the skills operative at high efficiency.

In life, however, numbers are rarely encountered except in situations which require that something be done

with them, the thing to be done depending on the type of situation. The school gives to the child 'problems'. These are to give him practice in meeting situations of the kind which he will find in life.

Thorndike[1] has presented evidence that the problems given in school are artificial in many ways and are not really comparable to life situations. But at present these problems seem to be the most satisfactory way of attempting to give life experience. With greater care in the preparation of problems many of the absurdities to which objection has been taken can be eliminated.

Taking problems as they are now used, many difficulties are encountered. When a problem is presented on the written or printed page, the child is at once under the necessity of reading, which is a skill presumably quite apart from his ability to solve problems. Nevertheless, if he is unable to read he certainly cannot solve the problem which might otherwise be quite within his power. Investigations have shown that a course in reading may greatly improve ability in arithmetic.[2] Probably a course in reading may allow an ability in arithmetic to become expressed which would otherwise be hidden. As far as school performance is concerned it may be said to improve the ability.

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1. Eg. The Psychology of Arithmetic - Thorndike p. 12.
2. W. W. Lessenger - Jour. of Ed. Research, 1925.

Teachers' opinions and experimental findings agree that the solution of a problem does not depend to any large extent upon the ability to perform the several numerical operations included. Of course, if an error in computation is made, the solution is wrong, although the method may still be right, and knowledge of the operations involved does not at all insure the correct working of the problem. That is, problem solving involves some ability or insight which is distinct from ability to master the fundamental operations. In an attempt to discover something about the ability or abilities here involved, much work in connection with problem solving is being done by investigators.

A first problem which one might set is this. To what extent does the ability to solve a problem using a certain operation lag behind, after the ability to perform that operation has been developed? Problems employing the same operation may vary in difficulty, but, a problem of even the simplest form is much more difficult than the operation it includes.

For a second problem one might try to discover how much of problem solving ability depends on instruction or acquisition of knowledge, and how much is dependent upon some innate factor which must be allowed to develop with growth before there can be comprehension of a process?

As a third problem one might study the order of events in a child's solution of a problem (or an adult's for that matter). Is there a definite analysis of the

situation before arriving at the method of solution, or does the child first seize the situation as a whole and then fit in the parts?

Hamilton[1] criticizes present educational practice in arithmetic saying that there is too much work on skills and not enough on the development of insight. Now whether or not this is true depends largely upon the answer to the second question asked above. If insight is a matter of development rather than education, it is useless to spend much time on it. But even if this be the case, it might be aided by a course of training, just as we believe that memory per se cannot be improved, though better habits of memorization may make it more efficient.

In connection with the last question raised, some quotations will be given. Dewey[2] says that "A conscious setting forth of the method logically adapted for the reaching of an end is possible only after the result has first been reached by more unconscious and tentative methods." Again Greene[3] in dealing with the steps used by a child in solving a problem states "This analysis and organization is not necessarily recognized by the child but it constitutes the second important step." The solution of this problem would have great effects on teaching methods. If the unconscious analysis and organization mentioned by Greene

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1. E. R. Hamilton - Jour. of Educ. Research, 1925.

2. J. Dewey - How We Think, p. 113.

3. H. A. Greene - Jour. of Educ. Research, 1925.

results from a well formed habit then we should be able to produce it by training. If it is unconscious because it involves some power which is not the result of training then we can do little for it. Again if this be true it appears that the steps of logical analysis which we attempt to get a child to follow in working a problem may be putting the cart before the horse. He may see a problem and its solution as a whole and divide it later into its parts and steps merely as a way of satisfying a teacher who has come to delight in logical presentations of material.

This chapter has indicated the nature of the problems which are to be encountered in the field of problem solving. The purpose of this investigation may be set forth as an attempt to gather some information with respect to them and more specifically:

(a) To inquire into the ability of school children to analyze problems ordinarily given them for solution.

(b) To inquire into their ability to understand general processes and relations apart from concrete situations.

(c) To determine whether the abilities mentioned above develop with the age of the child.

CHAPTER II

PRELIMINARY INVESTIGATION

The preliminary work in connection with this investigation was done with eleven boys, working with them individually. Two of these were seven years of age, three were nine, three eleven, and three thirteen. They were selected with the aid of their teachers to give one of low, one of average, and one of high class standing at each age level.

In an attempt to discover their methods of solving problems and the extent to which there was comprehension of the processes which they were using, these boys were given a number of tests.

They were first given a problem to be solved. After this had been done a number of slips were given them to sort. The slips had on them statements of three kinds - those that were true and needed in the solution of the problem, those that were true but not needed to solve the problem, and those that were false. The subject was presented with these in order as they had a bearing on the problem and was asked to place them in three piles as indicated above. The problem given was: "A man buys ice cream at $1.20 a gallon. From each quart he serves 5 dishes at 10¢ a dish. What is his profit on one gallon of ice cream?" The 35 slips will not be recorded fully here. Some, however, are of particular interest.

Those slips which included only processes to be employed or stated relations in a general way gave the most difficulty. They were placed incorrectly most times, and required the most consideration even when placed correctly. These will be noted.

24. When there is a profit the cost price is greater than the selling price.

25. There is a profit when the selling price is greater than the cost price.

29. The profit on a gallon of ice cream is found by taking the selling price from the cost price.

30. The profit is found by taking the cost price from the selling price.

Only the 11 and 13 year old boys and one of the 9 year olds were able to understand either the problem or the work in connection with it. Of these 7, three made errors in 24, 29 and 30, and two in 25, although they had worked the problem correctly. Questioning led to the belief that in most cases the correct answers had been obtained, not through an understanding of the relations of the quantities in the problem, but, at least in the cases of profit and loss, by a simple habit of taking the smaller number from a larger when the problem indicated that there had been a profit. The problem asked to find the profit. The numbers 2.00 and 1.20 appeared, hence subtract.

The other difficulties encountered were chiefly due to the possibility of working the problem in two ways. When

done one way some of the statements became not-needed, although they were needed for the solution which the series had been arranged to fit.

The difficulties encountered here led to the next type of analysis device. A four step problem was used and slips were prepared in connection with it, though in this case all were needed in its solution. The sixteen of them contained all the information necessary to its solution. These were arranged in a mixed order and given to the subject who was to place them sequently on the table as they had to be used in arriving at the answer to the question. The problem used was free from denominate numbers. "Alice is reading a book that has 240 pages. She has read 80 pages. How long will it take her to finish if she reads 20 pages in an hour and can read only 2 hours a day?" The slips when arranged read as follows: The book has 240 pages; she has read 80 pages; to find the number of pages yet to be read subtract (take away); the number of pages yet to be read is 240-80; 240-80 = 160; there are 160 pages yet to be read; Alice reads 20 pages in one hour; the number of hours will be found by dividing; the number of hours is 160 ÷ 20; 160 ÷ 20 = 8; she must read for 8 hours; she reads for 2 hours in one day; find the number of days by dividing; the number of days is 8 ÷ 2; 8 ÷ 2 = 4; she will finish the book in 4 days.

Here again the difficulty was particularly with those slips which had no numbers and referred only to processes

to be used. Other problems used indicated the same to be true, and it was this type of exercise that was selected for the later work in the school grades.

One other type of exercise was tried, however, which was of interest. This was to give a generalized problem to the subject and ask him to explain how it would be done. Then he was asked to make a problem, employing numbers, which was the same as the general one, and finally to solve a problem given, which was of the same type. A series of 15 problems was used, ranging in difficulty from very easy to fairly difficult. The first was: "Two boys each have some marbles. How do you find how many they have together?" The number problem with this was: "Jack has three marbles and Fred has four. How many marbles have they together?" Problem 13 proved most difficult and hence should have been last. "How do you find the area of a square when you know its perimeter?" "A square has a perimeter of 36 feet. What is its area?" Problem 7 was: "A classroom has a number of seats in it. If you know how many boys there are and how many girls there are, how would you find how many are still empty?" "A classroom has 13 boys and 12 girls in it. If there are 28 seats altogether, how many are empty?"

The result of this test was to show that ability in the solution of the specific problem went beyond the ability to explain how the general one could be worked. Subject 6 was able to do all three tasks at Problem 7, but failed to explain or give a problem of his own at Problem 8, though he

was able to solve the one given. Subject 9 did all three on 9 but only the number problem of 10, 11 and 12. In no case was the success as great on the general case as with the specific problem given. This seems to indicate that the understanding of a general principle, which is supposed to precede the ability to give it application in a specific case, does not exist in a way at least which may be given as an explanation by the child. In this individual work the subject was not limited to reading the problem stated in its general way. Careful verbal explanations were made by the experimenter in an attempt to get the subject to understand the situation, care being taken only to avoid the use of numbers. Subject 11, 13 years old, and with a high I.Q., was able to do the three tasks readily in connection with all the problems. However, some supplementary problems of greater difficulty were given him, and a stage was reached where he was unable to explain the problem apart from numbers, but was able to work the numerical problem given him.

From this individual work certain conclusions were drawn which led to the investigation following. The first of these was that analytic ability in ordinary school problems is a matter of age development, and as such is independent of the development in ability to solve the problems. The second was that ability to analyze problems does not guarantee that the problems will be correctly solved, even apart from mechanical errors. The third was that efficiency in solution of problems does not thereby mean that the solver

understands the problem, in the sense of appreciating the relations it contains, and of being able to express the problem situation apart from its specific case. Neither may he be able to understand it when it is expressed to him apart from its numbers.

CHAPTER III

METHODS, SUBJECTS, MATERIALS

Method

After the preliminary investigation outlined in Chapter II, it seemed advisable to extend the work done there over a large number of cases. This involved difficulties, for it meant working with groups rather than individuals, and so the possibility of close observation and questioning of a subject's procedure was impossible.

Problems of a relatively simple nature were prepared and with them sets of slips, 13 in each case, which included all the information and operations necessary for their solution. As well a number of questions on mimeographed sheets were given to each child to be answered.

Materials

Three-step problems which did not require knowledge of denominate numbers were used. The information needed was all present in the problems so that dealing with them involved only reading ability to detect the problems, and the comprehension of the relations necessary to solve them. The complete materials are set out below.

Problem 1

A girl is typing letters and has done 60 already. She is to type for 3 hours this morning and 4 hours this afternoon. If she types 8 letters an hour how many will she have done by the end of the day?

Analysis slips for Problem 1#

She has typed 60 letters already	(1)
She types for 3 hours in the morning	(2)
She types for 4 hours in the afternoon	(3)
To find how many hours she types in a day add 3 and 4 ≁	(4)
3 + 4 = 7	(5)
She types 7 hours during the day	(6)
She types 8 letters in an hour	(7)
The number of letters typed today will be found by multiplying 8 by 7	(8)
8 x 7 = 56	(9)
She types 56 letters today	(10)
The number of letters done will be found by adding what are already done and those done today	(11)
60 + 56 = 116	(12)
She types 116 letters altogether	(13)

The slips used were in size about 3/4 inch by 4 inches, with the wording put in one to four lines as necessary.

≁ In the later problems the slips indicating a process were expressed entirely without numbers.

Questions for Problem 1.

1. I added the 3 and 4 because ______________________________

____

2. I multiplied the 7 and 8 because _________________________

____

3. I added the 60 and 56 because ____________________________

____

Problem 2

John had 5 packages of fire-crackers. Each package had 20 fire-crackers in it. He used 10 of them each day for 6 days. How many fire-crackers has he left?

Analysis slips for Problem 2.

John had 5 packages of fire-crackers	(1)
Each package had 20 fire-crackers in it	(2)
Find how many fire-crackers he had by multiplying the number of packages by the number in each package	(3)
5 x 20 = 100	(4)
He had 100 fire-crackers	(5)
He used 10 fire-crackers every day	(6)
He used fire-crackers for 6 days	(7)
Find how many he used by multiplying the number he uses each day by the number of days	(8)
6 x 10 = 60	(9)
He used 60 fire-crackers	(10)
Find how many he has left by taking what he uses from what he had at first	(11)

100 - 60 = 40 (12)

He has 40 fire-crackers left (13)

Questions for Problem 2.

1. I multiplied 5 by 20 because ______________________________

____

2. I multiplied 6 by 10 because ______________________________

____

3. I subtracted 100 - 40 because _____________________________

____

Problem 3

John had 52 marbles. His father gave him 18 more. Then he lost 40. He shared what he had left evenly with his two brothers. How many did each of the 3 boys get?

Analysis slips for Problem 3.

John had 52 marbles (1)

His father gave him 18 more (2)

To find how many he had now add what he had at at first and what his father gave him (3)

52 + 18 = 70 (4)

He had now 70 marbles (5)

He lost 30 marbles (6)

To find how many he had left after losing some subtract what he lost from what he had (7)

70 - 40 = 30 (8)

He had 30 left (9)

There are three boys to share the marbles (10)

To find how many each boy got~~,~~ divide the number of marbles by the number of boys (11)

$30 \div 3 = 10$ (12)

Each boy got~~,~~ 10 marbles (13)

Questions for Problem 3.

1. I added 18 to 52 to find ______________________________

____

2. I subtracted 40 from 70 to find ______________________

____

3. I divided 30 by 3 to find ____________________________

____

Problem 4

The problem used in this case was the same as in Problem 3. The analysis slips were also the same. A different set of questions was used, however.

Questions for Problem 4.

1. Before John gave away any marbles to his brothers, which of these tells best what happened?

   a. the number increased, then it decreased.
   b. the number increased, then it increased.
   c. the number decreased, then it increased.
   d. the number decreased, then it increased.

2. What marbles did John share with his brothers?

3. With how many others did John share his marbles?

4. What does "share evenly" mean?

5. Upon what facts does the number of marbles that each brother received depend?

6. Tell what each of these numbers is or means in the problem or in working the problem:

52 ____________________

18 ____________________

70 ____________________

40 ____________________

30 ____________________

3 ____________________

10 ____________________

7. John had some money. He earned some more and then spent a few cents for a knife. He bought chocolate bars with the rest. Tell how you could find how many bars he would buy?

Problem 5

A girl is typing letters. She has 60 done already. She is to type for 3 hours this morning and 4 hours this afternoon. If she types 8 letters an hour, how many will she have done by the end of the day?

Analysis slips for Problem 5.

She has typed 60 letters already (1)

She types for 3 hours in the morning (2)

She types for 4 hours in the afternoon (3)

To find how many hours she types add the number in the morning and the number in the afternoon (4)

$3 + 4 = 7$ (5)

She types 7 hours during the day (6)

Note - The spacing of the questions above was such as to allow for plenty of room for answers.

She types 8 letters an hour (7)

The number of letters typed today is found by multiplying the number of hours by the number done each hour (8)

$8 \times 7 = 56$ (9)

She types 56 letters today (10)

The number done by the end of the day is found by adding what are already done and those done today (11)

$60 + 56 = 116$ (12)

She has 116 letters done by the end of the day (13)

Questions for Problem 5.

1. Why do you add 3 and 4?

2. How many letters did she type in one day?

3. Which of these describes the problem best:

   a. First you multiply, then you add, then you add.
   b. First you add, then you multiply, then you add.
   c. First you add, then you add, then you multiply.
   d. First you multiply, then you add, then you multiply.
   e. First you multiply, then you multiply, then you add.

4. Upon what facts does the number typed in one day depend?

5. Tell what each of these numbers is or means in the problem or in working it:

   60 ______________________________

   3 ______________________________

   4 ______________________________

   7 ______________________________

   8 ______________________________

   56 ______________________________

   116 ______________________________

6. A man knows how much oil he has in a tank. He also has two trucks with several barrels of oil on each. If he knows how many gallons are in each barrel, how can he find how much he has altogether?

The same materials were given to each of three grades, IV, V and VI, where possible in the same school. Problem 3 was given only to a Grade VI class.

In the classroom the procedure was as follows: The class was given the sheet of questions on which they first wrote their names, grade, and age. The reverse side of the sheet was then used for the working of the problem of which each had a printed copy. Time was given for all but the extremely slow to finish. The working of the problem gave an indication of the ability in solving written problems, but was also necessary for the subjects to become quite familiar with the problem before proceeding to the analysis work.

The list of questions and the worked problem were then put away, the packages of analysis slips having been distributed in the meantime. Before the pupils examined the slips, what was to be done with them was carefully explained. The problem - "John had 10 marbles. He gave 4 to Henry and 3 to Peter. How many did he have left?" - was put on the board. The class gave the items of this problem and the operations necessary which were also written on the board. Then these were arranged on the board in the order in which they were needed. Each child in this way had a demonstration of the procedure he was to follow with his slips. The packages were then opened and the class arranged them on their desks one under the other, until they were satisfied with the order they had used, or had given up in confusion. Ample time was given for most of the class to finish. A few,

however, did not complete the task. If they were unable to decide on any order in the time given it is altogether unlikely that they would have completed even with more time. The slips were then numbered from the top as they had them on their desks and replaced in packages again in that order. Slips which the child could not place were put on the bottom.

In the first three experiments a second package of slips was given at this stage and the same procedure followed except that, whereas in the first case they were urged to include all slips in their arrangement if at all possible, in the second they were asked to draw an "x" through and omit in their order any that did not seem necessary.

Having been arranged, the slips were put aside and the sheet of questions taken again. Each pupil wrote in his own answers to the questions. In Problems 1, 2 and 3 an explanation of what was wanted was made in the case of the first question, and the pupils were left to answer them without further direction. In Problems 4 and 5 the investigator read them one by one to the class as they answered them, without explanation, but with a reasonable time limit imposed. Otherwise with six or seven questions the slow ones would not have arrived at the later questions at all in the time available. The answer papers and slips were then fastened together with the paper-clip and collected.

Care was taken at all times to prevent one child copying or receiving hints from another and in all cases a minimum of explanation was given. In order to make the test

valid, however, it was necessary to make sure that all understood the nature of the task they were expected to do. Some individual explanation was given if a pupil appeared to have no conception of what he should do.

Subjects

The classes used for the various problems were:

Problem 1	Grade VI	Garneau School
	Grade V	Westmount School
	Grade IV	Westmount School
Problem 2	Grade VI	Alex Taylor School
	Grade V	Alex Taylor School
	Grade IV	Alex Taylor School
Problem 3	Grade VI	Alex Taylor School
Problem 4	Grade VI	King Edward School
	Grade V	King Edward School
	Grade IV	King Edward School
Problem 5	Grade VI	King Edward School
	Grade V	King Edward School
	Grade IV	King Edward School

The classes used were all considered normal - there had been no segregation of the better or poorer pupils of the school in any of them. In all cases the classes took a lively interest in the work they were asked to do, especially in connection with the sorting of slips which appealed to them much as a game, it being a novel way to them of dealing with an arithmetic problem.

A table is given here to show the number of pupils doing each problem and the other work in connection with it.

TABLE I

NUMBER OF PUPILS USED IN EXPERIMENTS

	IV	V	VI	TOTAL
Problem 1	38	34	42	114
" 2	42	43	33	118
" 3			35	35
" 4	43	38	33	114
" 5	44	40	37	121
TOTALS	167	155	180	502

CHAPTER IV

SUMMARY OF DATA

This chapter will include a number of tables and graphs which contain the results of the various tasks performed by the pupils. As there were over 500 cases it will be necessary to deal in averages and percentages and the individual answers are not recorded.

Tables follow showing the results obtained in the problem solving and analysis work of each problem. Problem 1, as summarized in the second table, proved the most interesting of the problems used. This was due to the fact

TABLE II

SUMMARY OF DATA FOR PROBLEM 1

	IV	V	VI
Problems attempted	38	34	42
Solutions correct	26	24	31
% Correct solutions	68	70	73
Analysis correct	7	15	24
% Analysis correct	18	45	60
Analysis correct and solution correct	6	11	16
% Correct analysis with correct solution	23	45	52

that the problem was solved correctly by the three grades with the same degree of success, the spread being only 5% between them. This is taken for our purposes as indicative of equal ability in the solution of this particular problem. One might suppose then that there would be equal, or at least almost equal, ability in analysis of the problem, if this be considered a necessary ability contributing to the solution. As shown in Table II, this is far from the case. The correct analysis in grade VI was 60% and that in IV only 18%. This is too wide a discrepancy to be explained by chance. Some factor must be operating and evidence will be brought forward later to show that it may be due to the increased age of the grade VI pupils.

Another unexpected situation is here shown. Those pupils who solved the problem are to be regarded as the best problem solvers for this particular problem, and hence the best at understanding its processes, and the best at analyzing it. When the analyses which they performed are examined, however, it is seen that in one grade they do better, in one the same, and in one poorer, than the class as a whole.

This situation has been found to exist throughout the investigation. Those who worked the problem correctly did slightly better on the analysis tasks than the classes as a whole, but not nearly as much better as would be expected by virtue of their greater ability in problem solving.

In this case it might be objected that the ability in analysis depends in a large measure on the ability to read the slips, and that reading ability is much greater in grade VI than in grade IV. But the language of the slips was no more difficult than that of the problem itself, and was taken directly from the problem as far as possible.

TABLE III

SUMMARY OF DATA FOR PROBLEM 2

	IV	V	VI
Problems attempted	42	43	33
Solutions correct	24	39	32
% Correct solutions	58	90	96
Analysis correct	3	11	23
% Correctly analyzed	7	26	69
Analysis correct and solution correct	3	11	23
% Correct analysis with correct solution	13	29	72

Table III offers some contradictory evidence for, in this case, all the correct analyses were made by those who had the solution of the problem correct. But in grades VI and V the number of incorrect solutions was very small, reducing greatly the possibility of there being incorrect solutions with correct analyses.

Table IV applies only to grade VI. As solutions in this case were all correct, a high degree of ability in the solution of the problem is indicated. The analysis was

TABLE IV

SUMMARY OF DATA FOR PROBLEM 3 (GRADE VI ONLY)

Problems attempted	35
Solutions correct	35
% Solutions correct	100
Analysis correct	14
% Correct analysis	40

NOTE: As solutions were all correct in this problem no difference exists between corrects and incorrects.

only 40% which again indicates that the ability to solve the problem does not depend on the ability to recognize the relations and facts in an isolated way.

Table V deals again with the three grades. The ability in problem solution is fairly close, being within a range of 20% while the ability in analysis jumps from 2% in IV to 54% in VI. This again seems entirely unwarranted if there is any close agreement between the analysis of a problem and its solution. If ability to analyze the problem is necessary for its solution, it is to be expected that when 34 pupils from grade IV solve it more than one should get an analysis of it. The method employed should

TABLE V

SUMMARY OF DATA FOR PROBLEM 4

	IV	V	VI
Problems attempted	43	38	33
Problems correct	34	34	33
% Correct	79	89	100
Correct analysis	1	6	18
% Correct analysis	2	16	54
Analysis correct and solution correct	1	5	18
% Correct analysis with correct solution	3	15	54

be accurate enough that more than one should be able to record his ability by it. What is shown by this evidence is that the solution of the problem does not depend on an analysis of it.

Table VI does not differ much from those already given.

TABLE VI

SUMMARY OF DATA FOR PROBLEM 5

	IV	V	VI
Problems attempted	44	40	37
Solutions correct	33	31	33
% Correct solutions	75	77	89
Correct analysis	3	12	16
% Correct analysis	7	30	43
Analysis correct and solution correct	3	11	16
% Correct analysis with correct solution	9	35	48

The individual results of the problems are given to indicate that the material given in Table VII, which is a compilation for all cases, does not hide in its averages wide discrepancies or variations which might appear in the individual problems.

TABLE VII

SUMMARY OF DATA OF TABLES II, III, IV, V, VI

	IV	V	VI	Total
Problems attempted	167	155	180	502
Solutions correct	117	128	164	409
% Correct solutions	70	78	91	81
Analysis correct	14	43	96	153
% Analysis correct	8	28	53	30
Analysis correct and solution correct	13	38	87	138
% Correct analysis with correct solution	11	30	53	34

The totals for the three grades show a situation much as has been seen in connection with the individual problems. Taken over all cases, the spread of ability in solution of problems used as indicated by correct answers is only 21%, from 70% to 91%. The range, however, is very much increased in the case of the analysis correctly done. The figures, when all cases are considered, are 8%, 28% and 53%. There is some ability which in the grade IV pupils is not yet present. Whether this is something which they have not yet been taught and which the VI's have learned

through two extra years of school life, or whether it is an innate ability which has not yet developed, it would be difficult to say. No reasons due to education are evident to account for this greater increase in ability to analyze, than in ability to solve problems through these grades. Further, any training which increased the one should increase the other. It seems tenable that this analytic power is one which comes with growth and age rather than with education.

When the analyses performed by those having correct solutions to the problem only are considered, the situation is not altered. The percentages of correct analysis go up slightly in the case of grades IV and V and remain the same in VI. The grade VI result is particularly significant. It includes a much larger number of correct analysis which makes the chance element much smaller, and further, it comes in the grade where the problem solving ability is highest.

The increase of ability through the grades is still very much marked, supporting the contentions in connection with the groups as a whole. But it serves to emphasize the belief that this ability in analysis is not very closely related to the ability to get correct answers in written problems. If it is, we should expect the best analyzers to be much the better problem solvers. This is not appreciably true in grades IV and V, and in grade VI there is no difference, that is, the poorest problem solvers do just as well on analysis as do the better problem solvers. This may be due, of course, to experimental error. There is shown, however, to be no relation of any significance between these two abilities.

An examination of the four cases in grade V, problem 1, in which there was correct analysis but incorrect solution, shows that these were not the result of mechanical errors, but of a failure to employ all the data of the problem. This indicates that in working the problem these subjects did not comprehend it fully. If it were a case of mechanical error, it could be contended that the analysis had been properly performed in connection with the problem, that, as a problem, it had been done correctly and that the errors in computation were incidental. However, such was not the case and the ability to analyze did not carry over to the working of the problem.

The one case in grade V, problem 5, is given here. The analysis was done correctly. The problem was solved as follows:

She has 60 done already.

She is to type 3 hours this morning.

She is to type 4 hours this afternoon.

Altogether she will type 3 + 4 = 7 + 60 = 67 (ans.) = 67 letters = 67 x 8 = 536 letters.

Ans. 536 letters.

His ability to analyze the problem did not prevent him from adding hours to letters. It appears that a possible explanation is that on reading over the question, there is formed at once a mental pattern of what is to be done with certain numbers, and that this is proceeded with without any critical examination. The numbers 3, 4, 60 and 8 appeared

and the pattern he got and followed through was:
$[(3 + 4) - 60] \times 8$ rather than $(3 + 4) \times 8 + 60$.
In question three of the question sheet he was able to select the correct series of operations when he had to examine them carefully. But it seems that in solving the problem this was not done. This might well be a case in which careful analysis on the part of the subject might have corrected his solution, but again this would rather have been a means of checking up on the plan already accepted, rather than an aid to the formation of a plan for solving the problem in the first place.

Analysis as considered above has been in terms of correct arrangement of the slips given in connection with the problem. Examination of the individual cases reveals that there are great differences in the qualities of the arrangements which have been scored failure. There are those which have but one or two slips misplaced; others have selected six or seven slips which they have arranged in a correct order leaving the rest unplaced; while in some there is no apparent order at all. In making comparisons it seems not altogether fair to consider all those who did not have a perfect analysis as equally deficient in analysis ability.

To compensate for this an arbitrary system of weights has been adopted, which will give different scores and perhaps a more accurate index to the relative abilities of the different grades in analysis, as measured by the slip sorting.

Four divisions, A, B, C and D, were made. Those of A quality have analysis correct or very nearly so, with only one slip misplaced. Division B includes all cases in which the ability in analysis is well marked, the errors being chiefly in failure to place the process slips, such as 3, 8 and 11 in Problem 2, correctly, or a number of inversions of order such as 2, 4, 3 and 11, 13, 12. Division C includes cases poorly done, but in which there is even slight evidence of arrangement, such as the grouping of the data slips, or the putting in sequence of the three slips which give the numerical operations. The cases in which there was no evidence at all of any comprehension of the task are classed in group D. The weights attached are 3, 2, 1, 0 respectively.

TABLE VIII

SCORES BY GRADES ON ANALYSIS WITH WEIGHTED VALUES GIVEN TO ARRANGEMENT OF SLIPS OF DIFFERENT QUALITY

	IV				V				VI			
	A	B	C	D	A	B	C	D	A	B	C	D
Problem 1	7	14	13	4	15	13	6		24	16		2
" 2	3	12	22	5	12	19	8	4	23	9	1	
" 3									15	15	5	1
" 4	3	10	20	10	17	15	7	1	19	12	5	
" 5	3	11	19	10	12	16	10		19	14		
Total Cases	16	47	74	29	56	63	31	5	100	66	11	3
Totals weighted	48	94	74	0	168	126	31	0	300	132	11	0
Possible Score	166 x 3 = 498				155x3 =465				180 x 3 = 540			
Actual Score				216				325				443
Percentage				43				70				82

Table VIII shows the distribution in the various classes by grades. The effect is to bring the scores much higher, for the failure cases contribute something. However, the general result is the same. Grade VI continues to be much above IV. In grade IV the cases of A or B quality are 37%, in V, 76% and in VI, 92%.

In the above summaries the problems with correct solutions have been isolated and dealt with. Table IX shows the analysis of those with incorrect solutions. The

TABLE IX

ANALYSIS BY SUBJECTS WITH INCORRECT SOLUTION

	IV	V	VI	Total
Problems incorrect	60	27	16	103
Analysis correct	1	5	9	15
% Analysis correct	2	19	56	14

number of cases is small, especially in grades V and VI. The same increase in ability to analyze is seen here as in the total groups and in the correct solution group. The ability to analyze shows marked increase regardless of the fact that the problems were not solved by any of these pupils. There are here 15 cases where the analysis work was correctly done but this ability did not at all insure the correct solution of the problem.

Three graphs are included here illustrating the conditions expressed in tabular form in Table VII.

A classification based on the solution of the problem and its analysis gives four classes:

1. Problem correct - analysis correct,
2. Problem correct - analysis incorrect,
3. Problem incorrect - analysis correct,
4. Problem incorrect - analysis incorrect.

Figure 7 illustrates graphically the distribution by grades in these various classes. The increases or decreases in percentages may be seen to be quite definite and uniform in all cases. The significant features have already been pointed out in connection with the tables but may be more readily discerned here. These are the great increase in 1, as indicated above, and the increase of 3 to the point in grade VI where the percentage of correct analysis is as high with those who did not solve the problem, as with those who did.

As indicated in Table VIII, the quality of analysis in the cases scored failure was not uniform. The slips included in the analysis process are of several kinds and it is possible that some may have proven more difficult to deal with than others. Class B was made inclusive particularly of those who had difficulty with the slips that have been called process slips, that is, those which indicate what operation is to be performed without reference to the numbers used in the problem. In the preliminary work it was found that the subjects made more errors with these slips

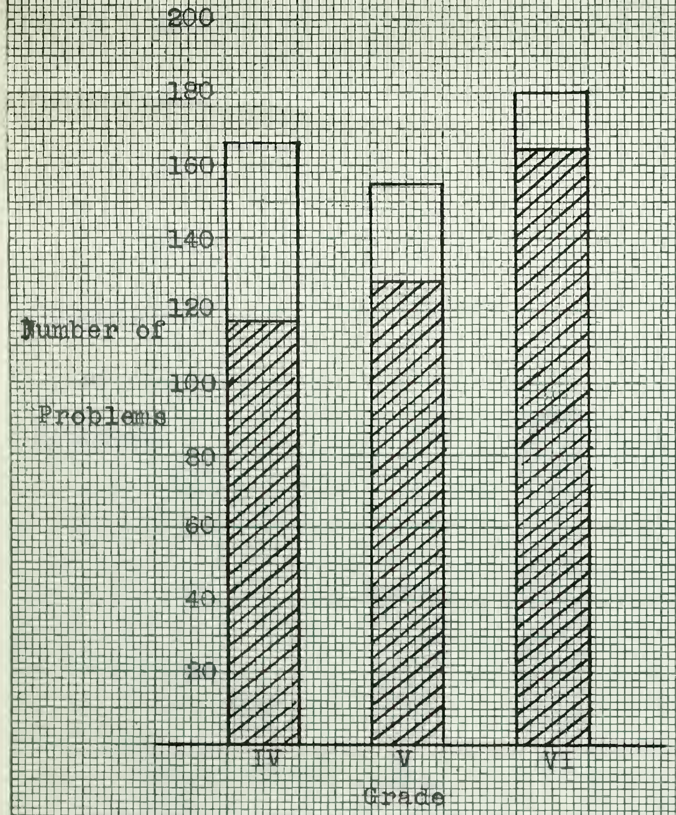


Fig. 1.- Pfoblems Attempted and Solved Correctly by Grades.
Total column - attempted
Shaded column - correct

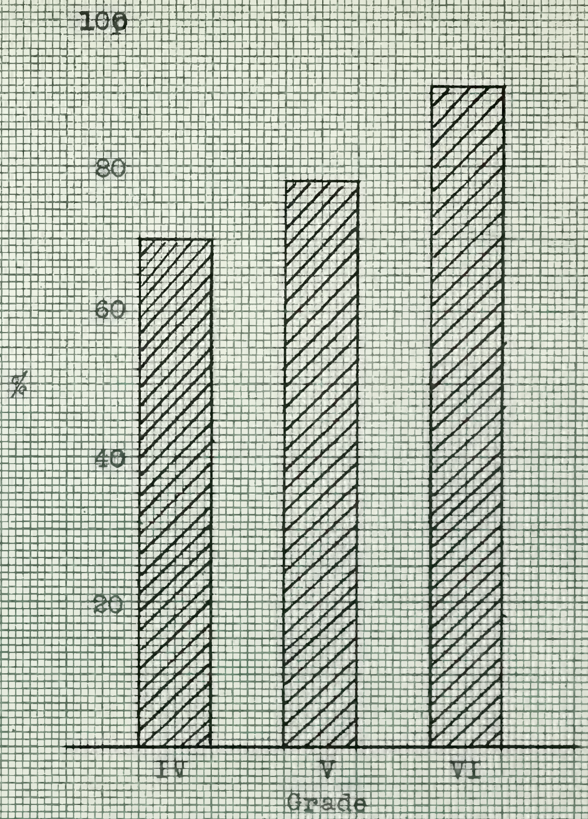
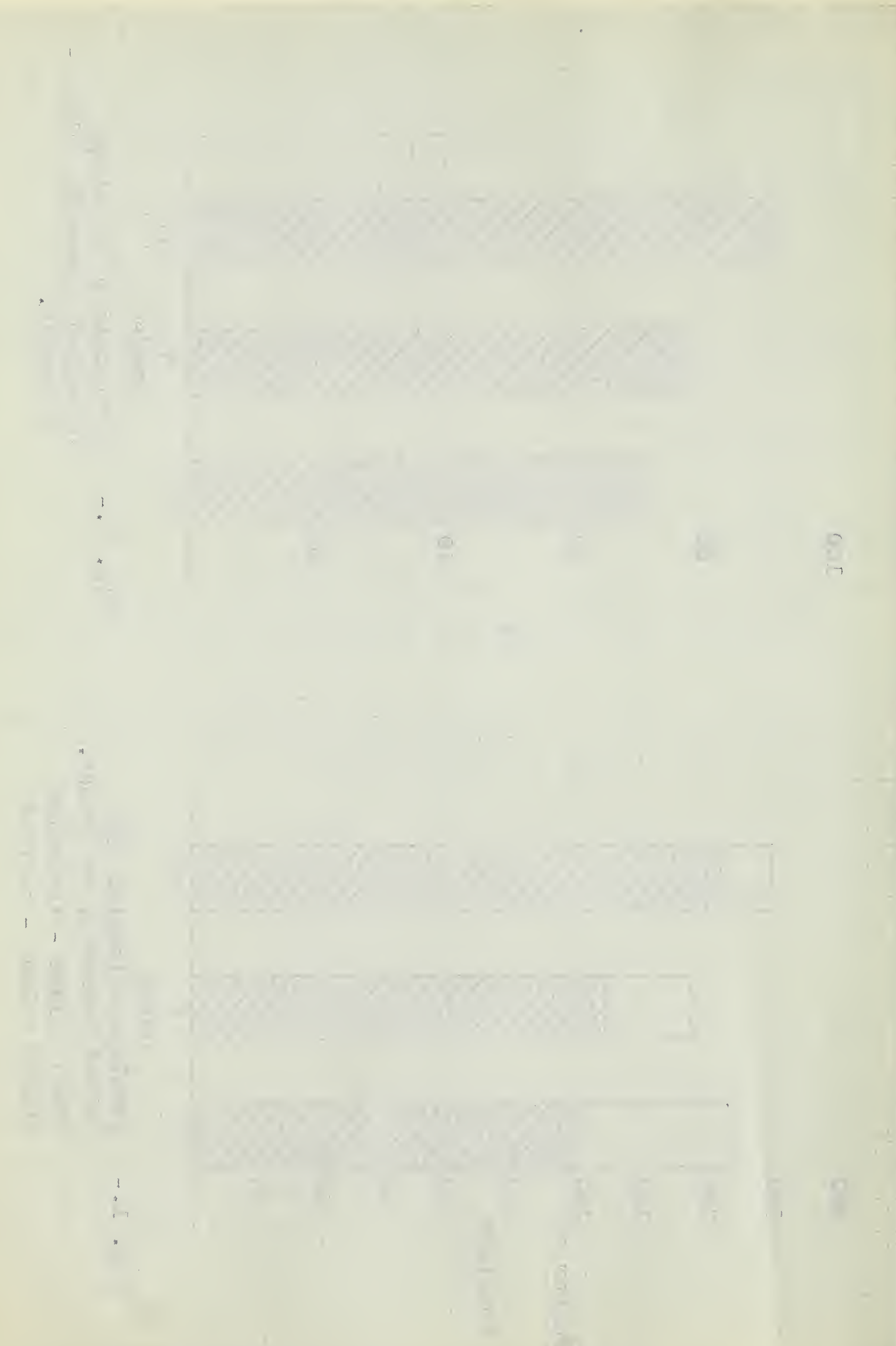


Fig. 2.- Percentage of Problems Attempted Correctly Solved - by Grades.



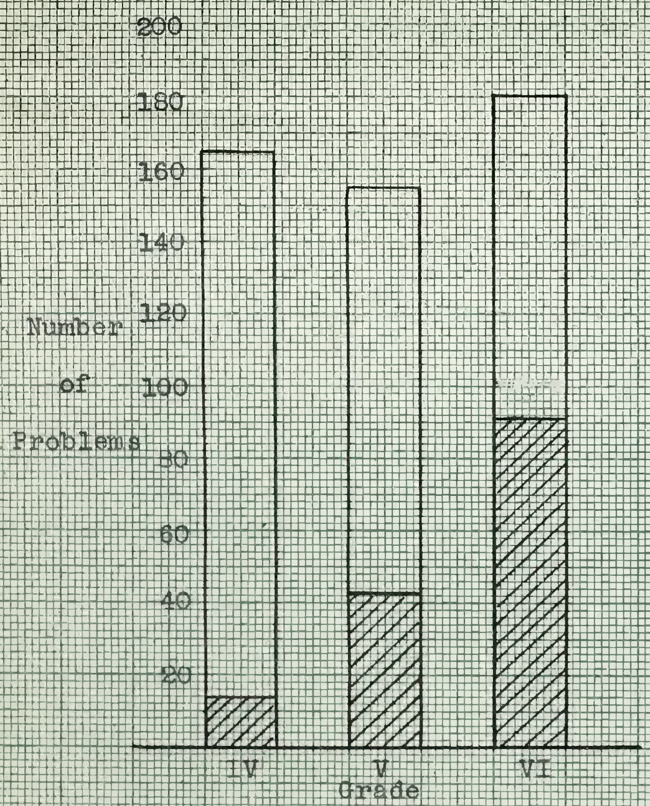


Fig. 3.- Problems Attempted and Problems Correctly Analyzed

Total column - problems
Shaded column - correct analysis

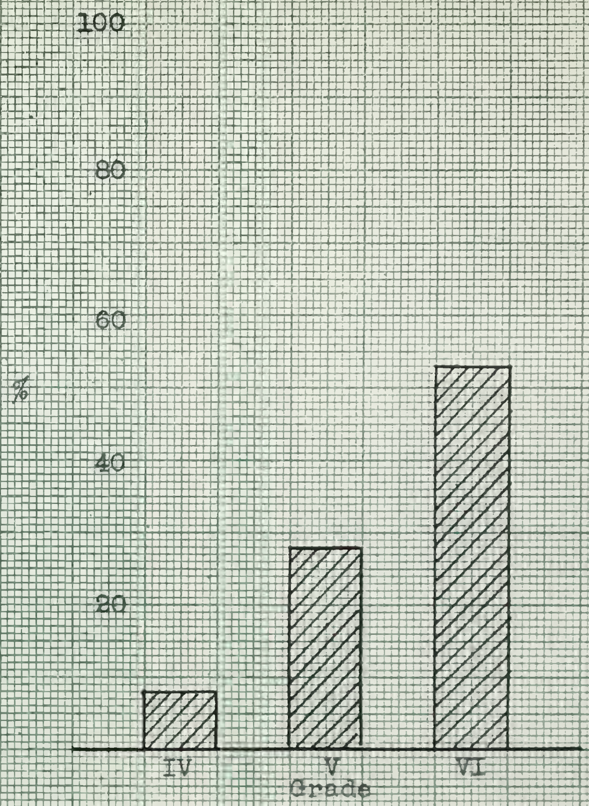


Fig. 4.- Percentage Correctly Analyzed of Total Number Attempted.

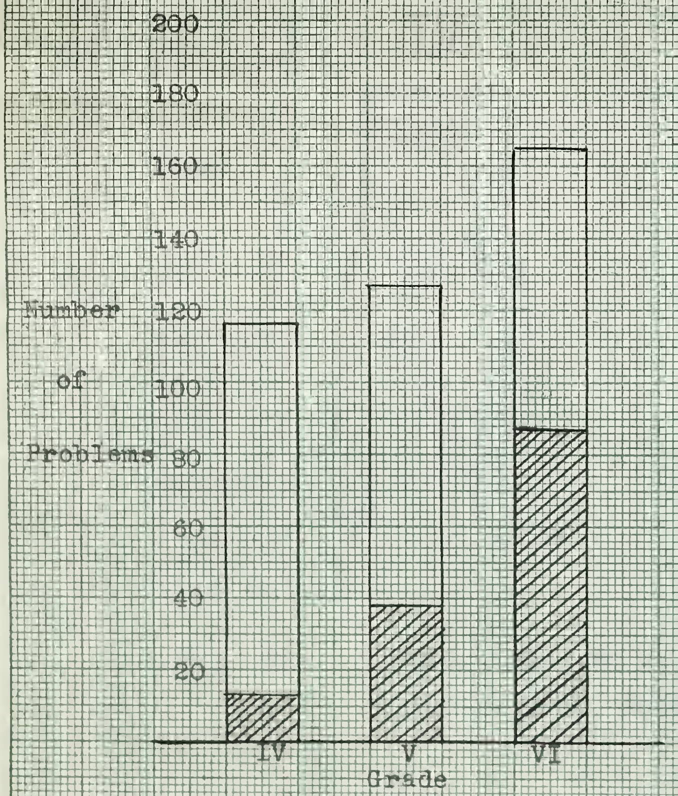


Fig. 5.- Problems Correctly Solved and Problems both Correctly Solved and Analyzed

Total column - problems correct
Shaded column - problems both solved and analyzed.

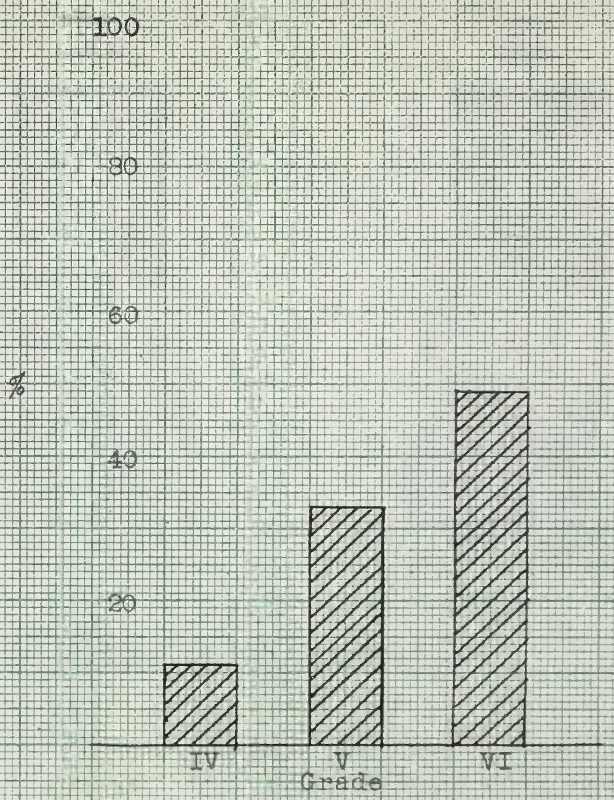


Fig. 6.- Percentage of Problems Correctly Solved Correctly Analyzed.

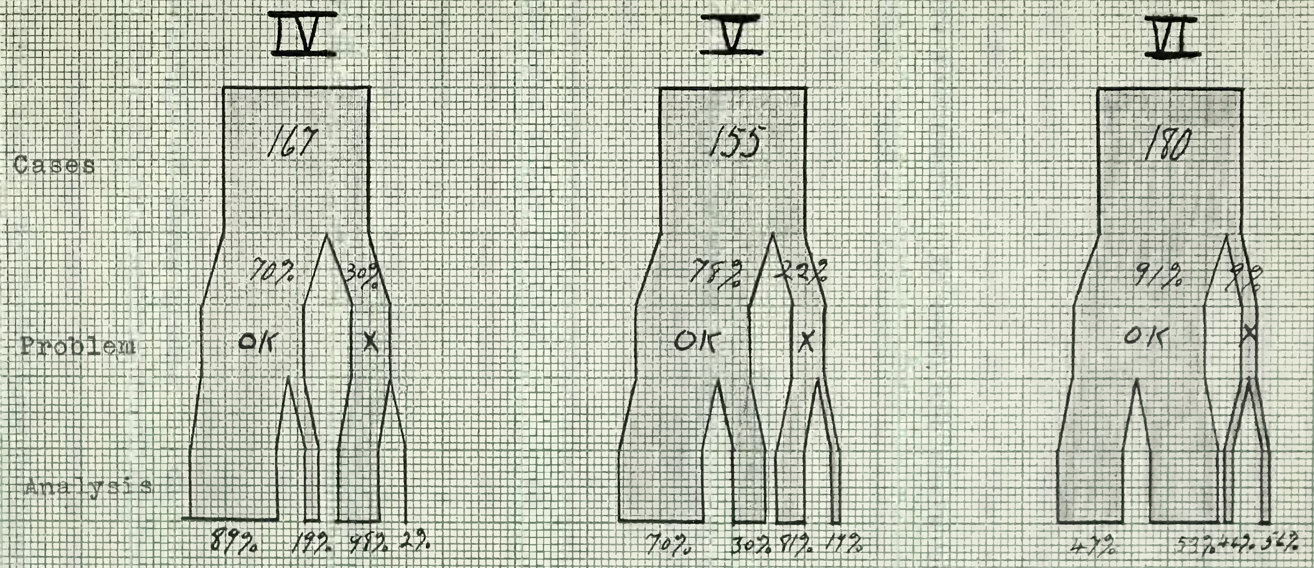


Fig. 7.- Distribution of Scores According to Problem Solution and Problem Analysis.

and spent much more time over them, even if they were placed correctly. Consequently it was thought advisable to determine to what extent these caused difficulty for the classes used in this experiment.

The first method employed to determine this was to give a second set of slips to the pupils and allow them to omit in their order any of them which they did not think were necessary. In each set there were three of these slips. Table X indicates the extent to which the pupils omitted them.

TABLE X

CASES IN WHICH PROCESS SLIPS WERE OMITTED ON SECOND ARRANGEMENT

	IV	V	VI
3 process slips omitted	10	22	31
3 process slips and others	16	13	23
2 process slips	6	4	6
2 process slips and others	1	6	
Totals	33	45	60
Total cases	80	77	112
Percentage	41	58	54
Percentage of three grades	51		

There may have been various reasons for the pupils omitting them. No doubt some followed the process through numerically with those slips containing the numbers and number relations and, finding this to give a continuous

series, did not feel a need for any statement telling, in a general way, what to do. Others probably could not understand the slips at all and for that reason omitted them. As it had been indicated that slips might be omitted there were some pupils who felt they must do so, even though they were cautioned carefully that they were still to retain them all if they seemed necessary to the solution of the problem. These no doubt crossed out the process slips as they could see readily that all those with numbers contributed something, while of the others they were not so sure. Many did include them all and consequently the percentages given in Table X are really lower than they would be if these were not considered.

When measured by the desire to omit them, the process slips caused much more difficulty than any others. As they were entirely generalized, knowledge of how to use them should indicate appreciation of the problem. They are the ones which indicate a knowledge of the problem situation itself apart from the specific case in which it happens to be presented. This is the ability which we presume to teach whenever a new type of problem is presented to a class. The general principles of it are taught either directly or through examples, and each subsequent problem worked by the pupil is to be an application of the general principle, and if the class is successful at working problems, it is because it understands the problem type. Now the evidence here seems to be that such is not the case, that this understanding is not at all necessary to work problems as presented by the

school, though those in life may present a different situation, and further, that such ability to understand general relations may be impossible until a definite degree of maturity has been reached.

Table XI deals with the sortings as a whole and is a

TABLE XI

CASES IN WHICH FAILURE WAS DUE TO
MISPLACEMENT OF PROCESS SLIPS

	IV	V	VI	Total
Failures in analysis	153	112	84	349
Errors in process slips	83	76	43	202
Percentage of errors in process slips	54	67	51	58
Cases with only process slips incorrect	28	42	28	98
Percentage of errors in process slips with no other error	33	55	65	49

summary of those cases in which the errors are definitely assignable to the process slips. The percentages are high, it will be seen, when all failures are taken into consideration. Also it must be kept in mind that in a great many cases it is impossible to say whether the error was due to a process slip or not. For example, if we find the sequence 9, 11, 10, 12, we cannot very well say whether the 11 has been placed ahead or the 10 back and such cases were omitted from the tabulation. In this table then it will be seen again that the process slips proved to be the ones causing the most difficulty.

The percentages in the last line of Table XI indicate less error in IV than in V or VI which at first glance seems to be a contradiction of what has previously been found and what would also be expected. However, it is to be noticed that grade IV has many more errors in analysis over all cases and this is just an indication that as well as having errors in the process slips that there were other errors as well.

The increases which have been shown to exist were taken on grade levels. Considering averages, this is to some extent indicative of age. The average ages of the pupils used in this experiment by grades were:

Grade IV	9.7 years
Grade V	10.7 "
Grade VI	11.7 "

This age rating should probably be increased by a half year as it is taken from averages of the ages given by the pupils in terms of years, and does not include the extra months.

The average ages given for the grades may be considered as roughly indicative of mental age, as the dull pupils are retarded and tend to be found in grades suited to their mental development, while those that are superior become advanced. Hence conclusions drawn with respect to grades might be considered as applicable to certain mental ages.

As each grade contained pupils of ages varying at least over five years, two years from the average above and below, cases are found through a wide age range, from 7 years to 15 years, though the numbers become small at the extremities each way.

TABLE XII
SUMMARY OF SCORES BY AGES

Age	Problem	Problems Attempted	Problems Correct	Analysis Correct	% Problems	% Analysis
7	1	1				
	2	1	1			
	3					
	4					
	5					
	Total	2	1	-	50	0
8	1	1				
	2	3	3			
	3					
	4	3	3			
	5					
	Total	7	6	-	86	0
9	1	23	15	5		
	2	19	9	1		
	3					
	4	25	22			
	5	23	18	2		
	Total	90	64	8	71	9
10	1	30	20	15		
	2	32	29	10		
	3					
	4	31	29	7		
	5	38	28	12		
	Total	131	96	44	56	33
11	1	35	28	19		
	2	35	29	11		
	3	16	16	6		
	4	21	17	7		
	5	21	16	4		
	Total	128	106	47	83	37
12	1	13	10	5		
	2	22	20	9		
	3	9	9	3		
	4	20	16	7		
	5	19	17	7		
	Total	83	72	31	86	34
13	1	10	7	2		
	2	4	3	2		
	3	8	8	3		
	4	10	10	2		
	5	11	10	4		
	Total	43	38	13	89	32
14	1	2	1			
	2	6	5	4		
	3	1	1	1		
	4	3	3	1		
	5	4	3	2		
	Total	16	13	8	81	50
15	1					
	2					
	3	1	1	1		
	4	1	1	1		
	5					
	Total	2	2	2	100	100

TABLE XIII

AGE SCORES WITH WEIGHTED VALUES
GIVEN TO ANALYSIS OF DIFFERENT QUALITIES

	A	B	C	D	T	P	S	%
7			2		2	6		
xW			2	0			2	33
8		2	4		6	18		
xW		4	4	0			8	44
9	14	31	31	15	91	273		
xW	42	62	31	0			135	49
10	49	42	31	8	130	390		
xW	147	84	31	0			262	67
11	46	51	21	10	128	384		
xW	138	102	21	0			261	68
12	34	31	17	1	83	249		
xW	102	62	17	0			181	72
13	16	18	7	2	43	129		
xW	48	36	7	0			91	70
14	9	3	3	1	16	48		
xW	27	6	3	0			36	75
15	2				2	6		
xW	6						6	100

A - Analysis correct - weighted x3
B - Analysis nearly correct - weighted x2
C - Analysis slightly correct-weighted x1
D - No evidence of analysis - weighted x0

T - Total cases for age.
P - Possible score - T x 3.
S - Score actually made - sum of frequencies x W.
% - Actual score of possible score.

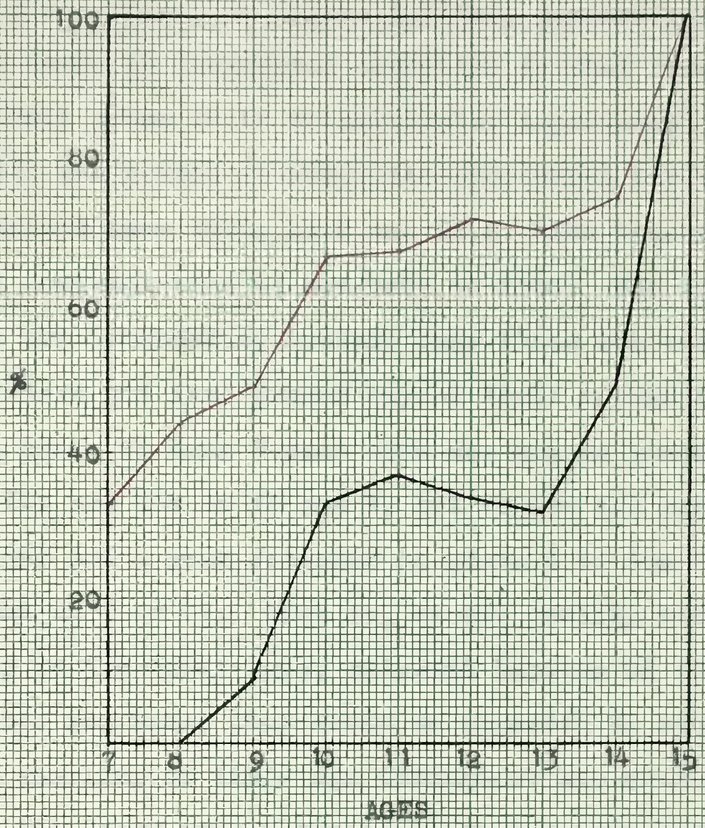


Fig. 8.- Ability in analysis at ages 7 to 15.

Correct analysis

Weighted analysis, according to quality.

Note.- Ages 7,8, and 15 have few cases.

Table XII is a summary of the results by ages. Table XIII deals with them similarly but ascribes to the failure cases in analysis weights as described following Table VIII. This information is also graphed in Figure 8. This summary presents what at first seems an unexpected condition in that from 10 years to 13 years there seems to be no particular improvement, while in dealing with the grades it was quite pronounced through what we should consider ages 10.2, 11.2 and 12.2. The reason for this apparent contradiction is not hard to find. The lower ages which we are considering include only those who through their ability have got into grades beyond their normal for their chronological age. We have 7-year olds who are in grade IV, 8's in grade IV, and 9's in IV and V, but we are not dealing with the 7's who are a year retarded in grade I or the 8's who are still found in I or II. That is, we are dealing with a selected group in the lower ages, selected because of their greater ability, and hence they would be expected to rank higher than a random sampling of their age throughout all grades. Similarly, at the other end of the scale we have the older pupils who are retarded. We have the 13-year olds who are a year or two behind but we are not dealing with those of the same age who have been advanced. Again we have a selected group, this time selected for their dullness, and as a result their performance is below that which would be expected for their age in general.

Tables XIV and XV have been prepared to show how this factor is operative. Dealing with those of ten years it is

TABLE XIV

SUMMARY OF TEN-YEAR-OLD DATA

	IV	V	VI
Attempts	43	69	19
Corrects	34	57	15
% Correct	79	82	80
Analysis correct	3	30	11
% Analysis correct	7	43	58

TABLE XV

SUMMARY OF TWELVE-YEAR-OLD DATA

	IV	V	VI
Attempts	8	27	48
Correct solutions	5	21	46
% Correct	62	77	96
Analysis correct	0	3	28
% Analysis correct	0	11	58

evident, as the number of cases decreases from IV to VI, that we have the upper end of the 10-year-old school distribution. Their analysis ability increases from 7% to 58% which indicates that the score made by our 10-year-old group is unduly high. If those from II and III were also present the score of 33% made in Table XI would be much reduced.

Again, the twelve year olds are seen to be increasing in number from grade IV to VI, which indicates that only the bottom portion of the distribution is present in the calculations. The analysis score also increases, and it is to be expected that if those in VII and VIII were present that it would go much higher yet. The score then, as indicated in Table XV for age 12, would be some advanced. This would also apply to Table XII.

Keeping this correction in mind a consideration of the graph, Figure 8, would indicate a more or less regular growth of the ability measured through the various ages of school life. The question again arises as to whether this is due to school training or a native factor. That there is some of each is doubtless true. It is evident in this case that the analytic ability has not kept pace with the problem solving ability and that the training which has been given in problem work designed to do two things, give ability in solving problems and understanding of general principles, succeeds much more in the former than in the latter. If training is the big factor we would expect the latter to parallel the former more closely. That it does not,

indicates that the mental-growth-with-age factor is high.

If, then, as the evidence would indicate, efficiency in the solving of written problems does not depend upon an ability to analyze them, upon what does it depend, for there must be something behind? Children certainly learn to solve problems. If it is not that they learn to analyze them, what is it that they learn? There is no evidence in this investigation that would indicate the answer to this question, but the alternative is that it depends on the formation of certain habits of response to words of the situation, which through repeated use in certain ways come to call forth responses. A course in problem solving becomes rather one in non-critical habit formation rather than critical analysis. This may not be the best policy, for it means that when problems are stated in an unusual way they may be apparently more difficult though the situation is not altered. Further, the problems of life do not come nicely stated with reverence for traditional form. It is often difficult to detect just what the problem is, and when detected it may be necessary to select from a mass of data that which has a bearing on it.

It brings us also to a consideration of the criticism so often made of education, that children are not taught to think. They are given ways of doing things which do not require that they be critical, and we find them unable to form opinions of their own. It must be remembered, however,

that there is a great deal of time and effort to be saved if the individual can surmount his difficulty by the use of a good set of habits which do not necessitate reflective thought.

But more important is the fact that as yet there has been produced no method for the training of thought. The power of thought is present and training it consists in giving it information with which to work, attitudes of criticism and perhaps a few devices whereby it may test its conclusions. The situation in connection with problems is analagous. Children are taught to solve problems rather than how to solve problems. That is, any general principles come incidental to the teaching of specific cases. Individual problems must be taught and as the capability develops within the child he is able to make his generalizations, rather the generalizations come upon him.

It is quite conceivable that, if the teaching of a problem type or principle were postponed until much later than it is now given in our schools, ~~that~~ the general principle might be taught and learned as such without difficulty. But under our present system the problems are presented and worked by the child in their specific form long before he is able to grasp their general principles. This is evidenced throughout mathematics. Many a grade IX pupil learns his geometry, in the sense that he is able to repeat propositions and make applications in certain exercises, without a comprehension of the significance of the principles involved. Later these become very evident to him.

There remains but one matter of additional summary for this chapter. Those for and against co-education are interested in comparisons between the sexes at various ages in doing various tasks. It is generally conceded that throughout the public school there is no appreciable difference in ability between boys and girls.

TABLE XVI

NUMBER OF BOYS AND GIRLS IN EACH GRADE WITH NUMBER OF PROBLEMS CORRECTLY SOLVED AND ANALYZED WITH THESE ALSO EXPRESSED AS PERCENTAGES OF THE TOTAL ATTEMPTS

	IV						V						VI					
	G			B			G			B			G			B		
Problem	C	S	A	C	S	A	C	S	A	C	S	A	C	S	A	C	S	A
1	21	15	5	17	11	2	21	13	8	13	11	7	14	9	10	28	22	14
2	24	10	1	18	14	2	20	17	7	23	22	5	16	15	12	17	17	11
3													18	18	10	17	17	5
4	24	17	1	20	16	2	23	17	6	17	14	6	18	16	9	19	17	7
5	24	20	0	19	14	1	22	19	3	16	14	3	18	18	13	15	15	5
Totals	93	62	7	74	55	7	86	66	24	69	61	21	84	76	54	96	88	42
% Correct Solutions		67			75			77			88			90			91	
% Correct Analysis			7			9			28			30			64			43

C - Number of cases
S - Number of correct solutions to the problem
A - Number of correct analysis of the problem

Table XVI summarizes by grades the scores of boys and girls by problems and grades. This information is more clearly set out in graphical form, Figure 9. It will be seen that the analysis ability coincides remarkably closely for grades IV and V

but that the ability of the girls takes a strong leap ahead of the boys in grade VI. This seems rather strange and no explanation can be satisfactorily given. It is worthy of note, however, that this happens when the average age of the girls is just at 12 years, about the time of the onset of adolescence, at which time girls jump ahead of boys in many ways. If this analytic ability is dependent upon development, it is not surprising that with girls and boys arriving at adolescence at different ages there should be a difference in their scores here.

The ages of boys and girls in the various grades is shown in Table XVII to be very closely comparable, with a slight margin in favor of the girls.

TABLE XVII

AVERAGE AGES OF GIRLS AND BOYS IN GRADES IV, V, AND VI.

	IV	V	VI
Girls	9.6	10.6	11.5
Boys	9.8	10.8	11.8

Table XVIII is a summary similar to that in Table XVI but with the scores weighted. The scores made in grades IV and V are remarkably close together while that in grade VI shows the same superiority of the girls. There is evidently some factor operating here. It can hardly be explained on the grounds that boys at this age are less willing to co-operate than girls and consequently might not have put forth

TABLE XVIII

SCORES OF BOYS AND GIRLS IN GRADES IV, V and VI
WITH ARBITRARY WEIGHTS
GIVEN TO ANALYSIS OF DIFFERENT QUALITIES

	IV				V				VI			
GIRLS	A	B	C	D	A	B	C	D	A	B	C	D
Problem 1	5	7	7	2	8	11	2		10	3		1
" 2	1	6	14	3	7	8	2	3	12	4		
" 3									10	6	2	
" 4	1	6	13	4	8	10	5		11	7		
" 5	1	8	11	4	7	8	7		13	5		
Total Cases	8	27	45	13	30	37	16	3	56	25	2	1
Totals Weighted	24	54	45	0	90	74	16	0	168	50	2	0
Possible Score	93	x 3	=	279	86	x 3	=	258	84	x 3	=	252
Actual Score				123				180				220
Percentage				44				70				87
BOYS	A	B	C	D	A	B	C	D	A	B	C	D
Problem 1	2	7	6	2	7	2	4		14	13		1
" 2	2	6	8	2	5	11	6	1	11	5	1	
" 3									5	9	3	
" 4	2	4	7	6	9	5	2	1	8	5	5	1
" 5	2	3	8	6	5	8	3		6	9		
Total Cases	8	20	29	16	26	26	15	2	44	41	9	2
Totals Weighted	24	40	29	0	78	52	15	0	132	82	9	0
Possible Score	73	x 3	=	219	69	x 3	=	207	96	x 3	=	288
Actual Score				93				145				223
Percentage				43				71				77

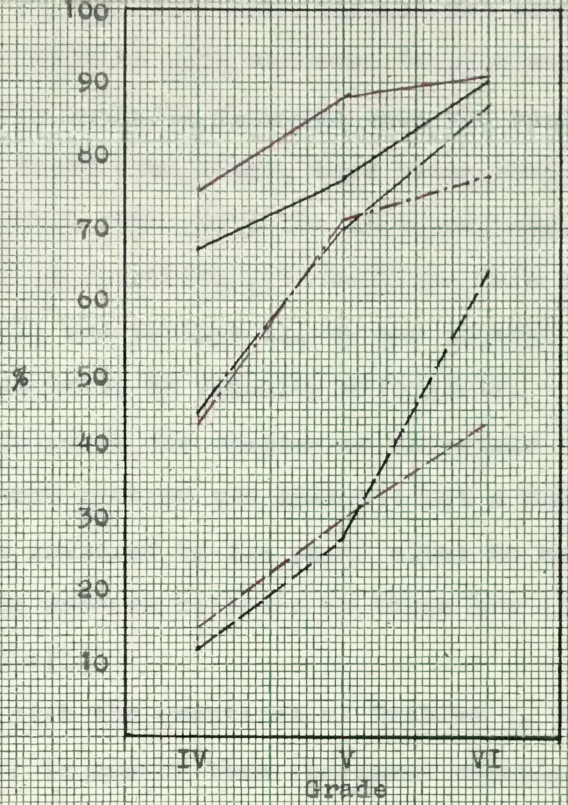


Fig. 9.- Percentage of Boys and Girls in Grades IV, V, and VI Successful in Solution of Problems and Analysis of Problems.

<u>Problems solved</u>
Boys
Girls

<u>Problems analyzed correctly</u>
Boys
Girls

<u>Weighted analysis scores</u>
Boys
Girls

their best effort, for they did equally well on the problem solving work. At this age there has not yet been removal from school to start to work and even if such were true of a few cases, they would be retarded ones who would tend to bring the boys score even lower, boys being removed more quickly from school than girls for slow progress.

It would be interesting to continue this study through grades VII, VIII, and perhaps IX, to see at what point, if at all, boys return to an equality with girls.

As indicated in the chapter on materials there was included with each problem and analysis set a number of questions to be answered by the pupil after he had done the other work. The answers received on these are listed in Tables XIX to XXIII.

Dealing with the questions of Problem 1, we find that the ability to answer them increased rapidly through the grades. This is true despite the fact that in this problem the solving was done almost equally well by the three grades concerned. The ability to answer these questions apparently had no marked effect on the problem scores obtained, which is in agreement with the conclusions in connection with the analysis slips.

In question 1 the preponderance of the answer "to get 7" in grade IV should be noted. This would imply that the child in solving the problem is not aware of his method. He adds the 3 and 4, not to get the number of hours worked, but to get the 7. He needs the 7 to work farther, it is true,

TABLE XIX

ANSWERS TO BECAUSE QUESTIONS - PROBLEM 1

1. I added 3 and 4 because	IV	V	VI
To find how many hours she worked	7	19	36
She worked 3 hours in the morning and 4 hours in the afternoon	2	6	4
I wanted to get 7	22	7	1
To find the answer	3		
To see how many letters she typed	2		
I thought you were supposed to	1		
To make it sensible			
You can't take 3 from 4		1	
To find how many she typed in the after'n		1	
I had to multiply 7 by 8			1
Omitted	2		
Totals	40	34	42

2. I multiplied the 7 and 8 because	IV	V	VI
To find the number of letters done in a day (or in 7 hours)	5	15	33
She worked 7 hours and did 8 an hour	3	7	7
To get 56	22	7	1
To find the answer	8		
To make sense	1		
You can't take 7 from 8		1	
To find how many hours		1	
To find how many she does in an hour		1	1
Omitted	1	2	
Totals	40	34	42

3. I added the 60 and 56 because	IV	V	VI
To find the number she typed altogether	1	12	32
She has 60 done and does 56 more	2	7	7
To get 116	17	6	1
To get the answer	17	5	2
To make sense	1	1	
To find how many she has to type today		1	
I wanted to know how many hours she worked		1	
Omitted	2	1	
Totals	40	34	42

and so his answer is quite appropriate for him. It has been suggested above that there is the possibility that a problem is analyzed only after there has already been formed a pattern for its solution. He has already solved the problem and, in being asked to analyze it, may work backwards. The answer is "to get 7." This is implying that the child is doing the natural thing because in grade IV he has not yet been trained through repeated teaching to supply a forward analytic explanation of what he has done. It is akin to the compositional method found so often among early school writers, of writing a composition and then making a plan for it after because one is demanded.

In this connection one of the subjects used in the preliminary investigation interchanged "the number of hours is 160 ÷ 20" (9) and "160 ÷ 20" (10) and said of them, "Well you'd think this (10) in your head first and put this one (9) down on your paper first." He believed he worked only with the numbers, knowing what to do with them apart from their connection with hours or anything else, and that he put down the statement first to satisfy the teacher.

It is often said that to have a problem well isolated or defined is to have it well on the way to solution. This isolation of it may be in terms of the method of its solution; that is, a knowledge of what the problem really is must involve knowing something of the nature of the

soluti/on more than just that an answer is to be found. We ask "Do you understand the problem?" If the answer is "yes", it implies that the nature of the answer is known and something of the process whereby it is to be arrived at.

Hence it may be that the answer "to get 7" given by the grade IV pupils is not as bad as it at first appears. After having worked the problem the child knows he must add 3 and 4 to get 7 to multiply by 8 to get 56 to add to 60 to get 116 which is the answer. What he wanted ultimately was the answer and he could not get it without the 7. The important point is that the child at this level does not consider his numbers in a problem in any generalized or abstract way. They are entirely tied up with actual numerical operations.

By grade V this answer type has become very rare, 7/34, and by VI has practically disappeared. The answer becomes "to find how many hours she worked." Just how much this answer really differs from the "7" it is difficult to say. It does show an ability in abstract expression of the same relation, though it has not resulted in appreciably greater problem accuracy.

Either answer implies that the direction of the pupil's thought is toward the result, not the information given. If, as true analysis would indicate, he starts from the given facts and works from them step by step, it would be logical to say that the 3 and 4 were added because she worked 3 hours in the morning and 4 hours in

the afternoon. But it is evident that the process must be teleological and the question is whether some type of analysis precedes the knowledge of the end of the problem being known. There is nothing in the results obtained here to indicate that such is the case.

In the teaching of problems there is an immediate difficulty in that it is impossible to teach an appreciation of a problem as a whole. It must be split up in the teaching. But there is no reason why this should not result in the other ability in the child. The point is that he will be more efficient in the solving of his problems the less of this he has to do in getting his method of attack. No matter what it is, it must include the elements of the problem but the relations need not be consciously set forth.

In looking over the solutions to the problems of the various grades, it is of interest to note that the IV's do not give statements. Their solutions are entirely numerical. Some solutions characteristic of grades IV and VI are given below.

<u>Grade IV</u>

1.
$$\begin{array}{r} 3 \times 8 = 24 \\ 4 \times 8 = \underline{32} \\ 56 \\ \underline{60} \\ 116 \end{array}$$

2.
$$\begin{array}{l} \quad\;\; 8 \\ \underline{\;\times 7 \qquad\quad} \\ 56 + 60 = 116 \end{array}$$

3.
$$\begin{array}{rcr} 3 & 8 \times 3 = 24 & 24 \\ \underline{4} & 8 \times 4 = \underline{32} & \underline{32} \\ 7 & 56 & 116 \end{array}$$

A great many of the IV's simply answer "She will have 116 done in one day" without any indication of the processes used.

Grade VI

1. In 1 hour she types 8 letters
" 3 " " " $3 \times 8 = 24$ letters
" 4 " " " $4 \times 8 = 32$ "
" 7 " " " $24 + 32 = 56$ "
She has 60 done already
In the day she will type $60 + 56 = 116$ letters.

Ans. 116 letters

2. A girl is typing letters
She is to type 3 hours this morning
" " " " 4 " " afternoon
" " " " altogether $4 + 3 = 7$ hours
In 1 hour she types 8 letters
" 7 " " " $8 \times 7 = 56$ letters
She has 60 letters done already. $56 + 60 = 116$

Ans. 116 letters

3. John had 52 marbles
John got 18 more marbles
70
He lost 40
30 left

Each boy got 10 marbles

4. John has 52 marbles
His father gave him 18 more
He lost 40 of them

$$\begin{array}{r} 52 \\ \underline{18} \\ 70 \\ \underline{40} \\ 30 \end{array} \qquad \begin{array}{r} 10 \\ 3\overline{)30} \\ \underline{30} \end{array}$$

The boys got 10 each

Again, many in the other grades work the problem through quickly and then as a matter of form write out the statements which they have been taught are necessary to a problem well done. It is as though they see the way it is to be solved and in fear of losing the thread if they stop to put down their statements of facts and relations, work it at once and fit the other on as an appendage. This work in statement of information is supposed to be an aid to correct solution. It would be worth testing out to see whether or not it is. Some pupils solve the problem with only a well-worded solution to show as their method. Under this conception these have a strong enough solution method in mind, that in spite of the distraction of putting down orderly statements, they are able to follow it through to the end.

A small number of cases in each grade gave the response that the 3 and 4 were added because she typed for 3 hours in the morning and 4 in the afternoon. In teaching, if the child were having difficulty, a most common question to put to him would be to ask how many hours she typed in the morning and afternoon, assuming that if the child knew or isolated the information he would know what to do. Such does not appear to be the case. If he knows what to do he will also know what to do it with, or will readily find out. What he needs is the end of the process, not the beginning of it.

Question 1 only has been dealt with here but examination of the other two questions shows a very similar set of responses.

Coming to Problem 2 the results are somewhat altered, especially in that the number of omissions or failure to

TABLE XX

ANSWERS TO 'BECAUSE' QUESTIONS - PROBLEM 2

1. I multiplied 5 by 20 because	IV	V	VI
To find how many fire-crackers he had	9	17	28
There were 5 pkgs. with 20 in each	8	16	4
To get 100	2	7	
To find the answer	4		1
5 is smaller than 20	1	1	
It's part of the problem		1	
You couldn't do anything else		1	
There is no 20x table	1		
Irrelevant	11		
Omitted	6		
	42	43	33

2. I multiplied 6 by 10 because	IV	V	VI
To find how many he used	9	16	26
He used 10 a day for 6 days	8	17	5
To get 60	2	7	
To find how many days	1		
To find what to subtract			1
To tell how many he used each day			1
Because it is smaller	1	1	
There is a 10x table	1		
To get the answer	1		
Irrelevant	8	1	
Omitted	11	1	
	42	43	33

3. I subtracted 100 - 40 because	IV	V	VI
To find how many were left	7	21	28
He had 100 and used 60	3	11	
To get 40	4	7	
To find the answer	2	1	2
To find how many he used			2
He had 40 left			2
You were supposed to			1
You can't take 100 from 40	1		
Irrelevant	5	1	
Omitted	20	1	
	42	43	33

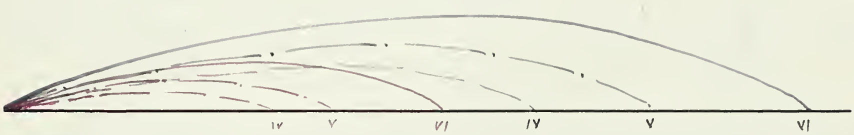
answer at all is greatly increased; in grade IV this class accounting for half the cases in question 3. This may indicate a strong language difficulty and reference to Table II shows that this grade dropped considerably in the solution of the problem.

Taken as a whole, over the three questions, the answers of the type "to find how many fire-crackers he had" give by grades 20%, 34% and 83% of all cases. With the solution of problems low in grade IV we would expect that those of ability in answering the questions in this way would be the better problem solvers. However, when these are taken alone, the percentage of such answers only goes to 31%. Apparently then the ability to state the reason for performing an operation in terms of its relations apart from numbers, which seems to imply a real grasp of what the problem is about, does not insure that it will be well done.

The converse seems true also, that ability to solve the problem, which should imply knowledge of it, does not mean that the pupil can tell about the processes involved which he has used.

There are, of course, various possibilities to be considered. The question itself and the type of answer expected may be beyond the language ability of the pupils. Insofar as this is true the measurement is not valid. It is difficult to get any simpler form of expression for the question and the child is left free to answer as he wishes.

It may be that the pupil is able to understand the generalization of the processes involved in some form of visual imagery which he is not able to verbalize. If so it cannot be measured. In any event this implies what appears to be true from the evidence given before, that the ability to deal in abstract relations is a much more difficult matter for the child than to solve the problem, that problem solving ability comes much earlier for any type of problem than the ability to appreciate the relations and principles involved in it. Illustrated schematically it appears thus:



The blue line represents the solving of problems while the red the comprehension of the principles involved. Although they both increase with grade, the latter is always behind.

Table XXI, dealing only with one class of grade VI, does not give anything of a comparative nature but serves to emphasize the degree to which the grade VI has become capable of expressing a relation apart from number. Eighty percent of the answers are of this superior type. It is to be noted that the score of this class on the solving of the problem was 100% (Table IV) and consequently there is no indication as to what their ability would be

TABLE XXI

ANSWERS TO 'TO FIND' QUESTIONS - PROBLEM 3.

	VI	
1. I added 18 to 52 to find		
To find how many marbles he had	27	
Because he had 52 and got 18 more	1	
To find 70	6	
To find how much his father gave him	1	35
2. I subtracted 40 from 70 to find		
To find how many marbles he had left	26	
To find 30	5	
How many he lost	4	35
3. I divided 30 by 3 to find		
To find how many each boy got	29	
There were three boys to share them	1	
To find 10	4	
To find how many I had left	1	35

or to what type of problem they would have to be taken to get a number of failures. If such were done it is very likely that the number of such answers would be much reduced. It is of interest in this table to notice that the answer next in frequency is that telling the answer to the operation,- to get 70, 30 or 10.

With problems 4 and 5 a different type of question sheet was used. Question 1, Problem 4, was intended to test the conception of relation. If it does this it shows only that ability to isolate the relations increased or jumped from relative inability in grade IV to considerable ability in grades V and VI. This does not seem entirely reasonable. The difference is due very largely to the fact

TABLE XXII

ANSWERS TO DIAGNOSTIC QUESTIONS - PROBLEM 4.

1. Before John gave away any marbles to his brothers which of these tells best what happened?

a. the number increased, then it decreased. x
b. the number increased, then it increased.
c. the number decreased, then it increased.
d. the number decreased, then it decreased.

	IV	V	VI
a.	11	26	23
b.	9	0	1
c.	2	6	5
d.	3	4	1
More than one marked	3	0	3
Omitted	15	2	0
	43	38	33

2. What marbles did John share with his brothers?

	IV	V	VI
What he had left	12	21	18
30	12	6	6
Data of problem repeated	4	5	1
2/3 of them	0	0	3
All	0	0	1
His own	0	2	2
1/3 of them	0	1	0
10 to each	9	0	0
20	2	0	0
Others (Wrong)	2	3	2
Omitted	2	0	0
	43	38	33

3. With how many others did John share his marbles?

	IV	V	VI
2	28	27	27
3	8	8	4
His brothers	0	0	1
None	4	1	1
Others (Irrelevant)	3	2	1
	43	38	33

4. Not significant.

5. Upon what facts does the number of marbles that each brother received depend?

	IV	V	VI
4 items of data (52, 18, 40, 30)			1
3 items of data (52, 18, 40)			1
2 items of data (marbles and boys)			1
2 items of data (omitting number of boys)		1	1
1 item of data	6	7	12
10	12	6	2
Human factors	1	2	1
Others	2	1	4
Omitted	22	21	10
	43	38	33

6. Tell what each of these numbers is or means in the problem or in working the problem: 52, 18, 70, 40, 30, 3, 10.

Possible good answers	301	266	231
Totals of good answers	220	212	212
Percentage of good answers	73	79	92
Omissions	40	18	1

7. John had some money. He earned some more and then spent a few cents for a knife. He bought chocolate bars with the rest. Tell how you could find how many bars he would buy.

Good explanation*	1	3	12
One item of data missing	3	3	
Money values supplied	8	1	8
Numerical answer given	8	5	1
Data repeated	1	1	4
Divide 5¢ into the money	1	2	1
If you knew what he had at first			1
He'd have none left		5	
Multiply by the number of bars		1	
Answer unintelligible		2	1
Personal factors - greediness, etc.		1	1
Omitted	21	14	4
	43	38	33

* Examination of the cases which gave a good explanation of the general case to determine their success in analysis shows the following result:

Analysis O. K.	6	2	0
" Wrong	6	1	1

It might have been expected that those who were able to do the former would be more able to do the latter. These few cases do not show any such relation to any high degree being little above the 30% mark for the whole group.

that 15 out of 43 IV's failed to answer at all, which may simply indicate a language handicap, that the words 'increase' and 'decrease' are not yet in their vocabularies. If that were the case they would not know how to deal with the question at all. However, reference to question 3, Problem 5, which is similar in form but in easier language shows the results to be very comparable, low in IV and equally high in V and VI.

There is some evidence here to show that the ability to understand the general relations develops in just such a way as has been shown in other parts of this experiment, and that it lags behind ability to solve a problem some considerable distance, and that hence it is not at all essential to working problems. It must be remembered, however, that this does not imply that such ability may not be of great value for other reasons and that if it can be improved by training that it is not of value to do so.

The slips of general relations in the analysis test were attempting to get at the same type of thing and gave a similar result.

The problem arises as to whether we should have children work problems to train them in analysis and generalization, or whether we should teach problems from the ability to generalize; i.e., if the child has certain abilities in analysis of situations, to see causes and effects, that working on this basis we can train him to solve problems. The former is rather the case according to the evidence here.

If we regard the function of an arithmetic course as being the training of children to think - to adapt themselves to new situations, and particularly the arithmetic ones to be found in the world of affairs and business - then the best conception upon which we may work is that abilities have a gradual development and we may improve their efficiency through practice, though we cannot produce them.

In evaluating the results of the answers on this question it is difficult to say how much of the various factors enter in. The relation of problems correct to increase in diagnostic question ability tends to the belief that the native development factor is high. If it is a language difficulty the problem discrepancy should not be so great, for the language of the problem is equal in difficulty to that in some of the questions. The question "Why do you add 3 and 4?" is certainly not one to cause the child difficulty because of its language.

The implication is that it is a waste of time to try to get too much generalization on the part of children early in the school program, but that they can be given problems to work which seem to require an ability in comprehension much beyond that which can be found within their capability.

Question 2 proved rather too easy to be of much value in showing differences. When the answers which are reason-

ably permissible, such as 'what he had left,' '30,' '10 to each,' and even '2/3 of them' and '20,' are taken out, there are not many other cases.

Similarly with question 3, the score in all grades was high, though with a slight margin in favor of VI over V, and V over IV. This is hardly more than could be accounted for on the grounds of better reading ability and when the relative abilities on the problems (Table V) are considered it is seen that the IV's have easily kept pace with their record here. The question is not diagnostic because it does not demand an answer requiring any type of abstraction; it can be answered by a direct reading of the number from either the solution which the child has given, or from the analysis slips with which he has been working.

The answers for question 4 have not been recorded as there were practically none which did not indicate a knowledge of the term. Evidently cases of failure in this problem were not due to a misunderstanding of the expression 'share evenly.'

Question 5. This exercise is highly dependent upon the comprehension of the words "facts" and "depend." It may be that language is the thing tested. The results are interesting. The answer to be expected, if the situation is fully grasped, is that the number of marbles a boy receives will depend on the number to be distributed and the number of boys to be divided among. This answer, however, did not appear in any grade. The number of omissions was high, and the number of responses indicating that the

question was not understood was also high, but there were many cases in which there must have been an understanding of the question to give the answers recorded. These are the cases in which one or more items of the data of the problem were given, facts upon which either directly or indirectly the number a boy received did depend. The answer to the problem, 10, the amount each boy did receive, was given in many cases which indicates that the pupils did not know what was wanted but only that it had something to do with what each boy got.

Assuming that any amount of comprehension of the problem would be reflected in the giving of some of the conditions of the problem - data - the following summary is included:

IV	V	VI
6/43 (14%)	8/38 (21%)	13/33 (40%)

This shows that the VI's at least understood the question much better, though in this case it is not so clear that it depended on seeing better the relations of the problem. It is a possible factor.

Use of a number in a problem involves a knowledge of what it means or represents in the particular situation. The series in question 6 was designed to determine to what extent there was comprehension of the meanings of the numbers used. The results show that on the whole there was

good understanding, though the grade increase is evident, comparable to that in success in solving the problem.

	IV	V	VI
Explanation of numbers	73%	79%	91%
Problem solution	79%	89%	100%

In each case it will be seen that there were some who were able to work the problem who could not give an explanation of the numbers which they had used. This seems also to be evidence that comprehension of what is being done in solving a problem lags behind the ability to solve it. This exercise would appear to be quite a simple one as far as the language of it is concerned and careful explanation was made so that all should know what was required. After having used "30" in arriving at the answer, and having sorted the slip which contained this information, it would be expected that the grade VI pupils, who had made a perfect score in problem solving for this problem, should have been able to explain in a simple way what the number stood for. Inability to do so seems to mean that the number must have been employed in the solution process without any significance having been attached to it. Is this desirable or is it not? And in either event can it be avoided? The result of the various information gathered here seems to be that it cannot, that comprehension is impossible beyond a certain stage at any grade level.

Given orally and with opportunity for individual diagnostic questioning there is good reason to believe

that the type of question given in #7 is an excellent means of determining whether or not the child understands the process of a problem. Thorndike in New Methods in Arithmetic recommends it as a class exercise.

The preliminary work shows that the ability to do this kind of problem solving is much less than it is for corresponding specific problems. It is easier to think or organize clearly

"Add 18 and 52, take away 40, divide 30 by 3" than to do the same with

"Add the money John has now and the money he earned, take away what he spent for a knife, divide what he has left by the price of a chocolate bar."

It may not be that the task is intrinsically harder, but just that it involves more words - comparable to the memory span for digits or syllables. It requires a type of syncretization which may be choked by too much bulk.

The results obtained are:

IV	V	VI
1/43	3/38	12/33

The ability is evidently well marked in its increase. There is no doubt but that those who were able to make a good response to this question were able to see the problem in its abstract relations. That all those who could not make a good response were unable to do so is not a justifiable conclusion, but with the discrepancy so large between

grades VI and IV it is a warranted conclusion that the ability of grade VI pupils to do a generalized problem of certain difficulty is much in advance of that of grade IV even when their difference in ability in problem solving is considered. That is, grade VI does better on problems than does grade IV, but this superiority is not nearly so much as that in doing the generalized case.

Omissions of the question were VI - 4, V - 14, and IV - 21. This, as expressed before, may be due to an inability to understand the language used, but must in many cases be due to failure to comprehend the situation, to inability to hold all the necessary factors in mind and organize them.

The answers to the questions of Problem 5 are given in Table XXIII.

TABLE XXIII

ANSWERS TO DIAGNOSTIC QUESTIONS - PROBLEM 5

1. Why did you add 3 and 4?	IV	V	VI
To find how many hours she typed	22	29	33
To get 7	5	3	
Because she typed 3 hours in the morning & 4 hours in the after'n	4	6	2
To find the answer	4		
Omitted	4		
Other answers	7	1	2
	44	40	37

2. How many letters did the girl type in one day?

	IV	V	VI
56	5	12	11
116	37	25	24
8		1	1
7		1	1
192	1		
60		1	
Omitted	1		
	44	40	37

3. Which of these describes the problem best:

a. First you multiply, then you add, then you add.
b. First you add, then you multiply, then you add. x
c. First you add, then you add, then you multiply.
d. First you multiply, then you add, then you multiply.
e. First you multiply, then you multiply, then you add.

a.	8	3	2
b.	16	34	31
c.	10	2	1
d.	6		1
e.	2		2
Several	2		
Omitted		1	
	44	40	37

4. Upon what facts does the number typed in one day depend?

On the number typed per hour and the number of hours			3
On the number of hours	1		6
On the number per hour	2	5	2
Numerical answer	4	3	5
Personal factors - how hard she worked, etc.	5	2	3
Others	5	6	16
Omitted	29	24	12
	44	40	37

5. Tell what each of these numbers is or means in the problem or in working it: 60, 3, 4, 7, 8, 56, 116.

Possible correct answers	308	280	259
Totals of good answers	212	214	241
Percentage of good answers	68	76	91
Omissions	41	0	0

6. A man knows how much oil he has in a tank. He also has two trucks with several barrels of oil on each. If he knows how many gallons are in each barrel, how can he find how much he has altogether?

	IV	V	VI
Good explanation			4
Add what is in the tank to that in the barrels	3	5	17
Numbers supplied	5	2	4
Multiply the gallons per barrel by the number of barrels		5	
Add	16	13	3
Multiply	1	4	3
Others	10	5	2
Omitted	9	6	4
	44	40	37

Question 1 is similar to that in Problem 1, question 1. The answers were somewhat different. The number of cases answering "to get 7" is much reduced while in V and IV the number answering "to find the number of hours typed" is greatly increased. The ability to answer in this form is still much more marked in VI than in IV. In the case of grade IV only one-half the cases gave this reason which implies conscious knowledge of what is being worked towards when the operation is performed. Three-quarters of the cases solved the problem. These others must have had some reason for performing the operation that they did, but if ~~this~~ apparently was not consciously this one. And it seems that in many cases where this answer was expressed that the operation was performed and that having been done there was realization as to why after rather than before. This much is evident, that it is not necessary to be able to give the

general relations of the problem to solve it. If it is not necessary, the suggestion made above that the explanation is made after rather than before the operation becomes an even greater possibility.

Question 2 might be regarded as a test in reading ability, for it demanded a careful differentiation of quantities. The answer given in the greatest number of cases in all grades was 116, the answer to the problem, but incorrect for this question. The question asked for the number typed in one day, which had been worked out to 56 by a process which should have been understood. The 116 was the number of letters done by the end of the day, including 60 done at some time before. The question, or perhaps the problem, may have been slightly ambiguous, but having been worked to the correct solution through the correct steps, and using the numbers for the right operations, it is to be expected that the pupil should know what each number represents. If he does not, and he did not in a great many cases of correct solution here, it must mean that the problem was solved to a large extent without a comprehension of what it meant, or why the operations were performed. It is known that in the lower grades, where problems tend to be much more stereotyped in form, that the key words such as "altogether" or "how many more" give a great deal of clue as to what operation is to be used with the numbers present. In a problem such as we have in this case it is not

so obvious that specific words should indicate what is to be done. Yet something akin must be the method used if the child is not aware of just what he does or the meaning of the numbers with which he works.

The correct replies did show an increase through the grades between IV and V - again out of proportion to the success on the solution of the problem.

In question 3 there was an attempt to do the same kind of thing as in Problem 4, question 1, but the language was made easier to avoid some of the failure there which seemed attributable to the fact that the words "increase" and "decrease" were beyond the vocabulary of many of the pupils. The words add and multiply were substituted as indicating the relations involved through words with which, through constant classroom use, the children should have been familiar. The child had already solved the problem and employed, at least in the cases of correct solution and in some where there were only mechanical operation, the processes indicated.

This problem employed three operations and to arrive at the correct answer they could only be used in one order. The results obtained were:

	IV	V	VI
	36%	85%	84%
(Cf. Problem 4, Question 1) →	25%	68%	70%

In view of the fact that the language is much simpler than in Problem 4, question 1, it is surprising that the success was not much greater. This gives reason for believing

that failure was due, not to the difficulty with the words used, but with the actual comprehension of the processes which had been used. As there was only one case of failure to reply it appears that all at least thought they understood.

A much larger number of these pupils had used the processes correctly than indicated the relations in this way, for in IV there was three-quarter accuracy. They used the right processes, but when confronted with the possibilities were unable to say what they had done.

It is well established that there is greatest efficiency in the solution of any problem when it is possible to perform the operations without the necessity of reflective thought. In using the formula πR^2 continued practice enables the user to make the substitutions and perform the operations without much reference to the significance of the formula. At the same time it is generally believed that this efficiency is really based on a knowledge of the processes of the problem and that these could be explained if the solver were asked to do so. With the pupils here studied this does not seem to be the case. They are able to perform operations with numbers in the situation but apart from the numbers are unable to say with any degree of certainty what the operations are. And the ability to do so appears to be a function of the mental age as represented by grade.

In question 4, as in Problem 4, question 5, the language difficulty is high. The VI's understood the question better

as shown in their answers but this may be because they knew the words "facts" and "depend." Fact is a small word but its meaning involves a mental grasp considerably developed. At any rate the increase in ability through the grades is pronounced - 7% - 13% - 30%. This includes all cases in which any of the essential data were given. Three cases of VI alone gave an answer fully correct - the number typed per hour and the number of hours worked. There is evidence here to confirm the belief that this ability is much behind that for problem solving, and that it develops more rapidly through the grades than does the former, at least through IV, V and VI.

It is apparent that this question taken alone has not much value because of the difficult language employed, but it contributes something when considered with the other evidence. It is certainly not negative and may be positive.

Omissions are again high. Two-thirds of the IV's, over one-half of the V's, and one-third of the VI's did not answer.

The comments made with reference to Problem 4, question 6, apply to question 5, for the results were very similar. Good answers were given in 68%, 76%, and 91% respectively. Compare this with the correct solutions of the problem: 75%, 77%, and 89%. The number of correct solutions is above that of explanations of the meanings of the numbers used. If solution of the problem involves comprehension of it, (and knowledge of the meanings of the

numbers should be one of the simplest forms of its comprehension), than all those who got it correct should have explained the numbers, and the percentage success would have been equal to that of the solutions. But, also, some of those who failed should have been able to explain at least some of the numbers, those which might be read directly from the problem, to which they were allowed to refer as the questions were being answered, and the success should have been much greater. The conclusion is that a great many, even more than is indicated by the above percentage discrepancies, who solved the problem did not know at all the significance of the numbers with which they were working, and hence did not understand the problem. Ability to work the problem does not imply comprehension of the principles involved in it.

With reference to particular numbers three are recorded:

	IV	V	VI
60	79%	92%	100%
56	50%	48%	81%
116	52%	35%	78%

The low scores in IV and V are seen to be in the case of 56 and 116. This bears out the low score in question 2 for the number typed in a day. Neither from the reading of the problem nor from its working does there appear to be a true understanding of the relation of those typed today, previously, and altogether. Notwithstanding this, it is possible to work the problem correctly - only 35% of the V's

could tell what the 116 was when they had it, but 77% were able to get it, and the explaining was done after the problem had been worked and the slips analyzed which would have been expected to increase the knowledge of the problem.

Question 7 did not produce as good results as its corresponding one in problem 4. There were no successes in IV or V to compare in any way with the VI's. However, in VI there were four good explanations, indicating that the ability had developed to some extent in this grade. An index for comparison may be found, however, by considering another response, "add what is in the tank to what is in the barrels." This is correct as far as it goes, but it omits to explain how the amount in the barrels is to be determined. The IV's produced 3 cases, V's 5, and the VI's 17 so there is some basis for comparison. Grade VI is again very much in the lead, and again far more than the extra ability in problem solving would warrant. Again this ability in generalization seems to be much dependent upon age. An interesting experiment would be to attempt to discover how much in the way of explanation of general problems could be obtained from a child by continued exercise on this type of problem and systematic teaching of it. The evidence here points to its being quite limited.

Everyone can recall in his experience inability to understand some process in his early school career which later seemed absurdly simple although no practice of any kind can be recalled as having intervened.

CHAPTER V

CONCLUSIONS, CRITICISMS, SUGGESTIONS

Conclusions:

The evidence produced in this investigation points to the following conclusions which have been dealt with in various places in the body of the report:

1. The ability of school children to understand problems lags far behind their ability to get correct solutions for them.

2. An ability in analysis of a problem is not essential to efficiency in its solution. The solution depends rather on certain habits of response which do not require analysis.

3. Such analysis of a problem as may be performed by a child, either in explaining how a problem was worked, or in setting down statements regarding it in his solution, may be rather in explanation of what has been done by more subtle and non-understood means, than in aid of the solution of the problem.

4. There is present an innate factor which determines the limit to which a child may deal in abstract relations and which increases through the public school grades - at least IV, V, and VI - and hence must be closely linked with age or mental age.

5. The ability in explanation of principles involved in a problem does not greatly increase efficiency in its

solution, nor does inability at all imply failure on a problem.

Assurance on these conclusions would mean some change in school aims and methods. It would mean that there would be less attempt to get from children at early ages evidences that they had abstracted from their work in arithmetic general principles which they could apply in other situations. At present we say if you can do A you should be able to do B because the general principles are the same. The evidence here, however, indicates that the pupils can very well do A without understanding what is involved in it, and hence could not do B until taught how to handle its specific cases. Thorndike[1] in this connection says, "It is assumed that all problems are and should be solved by some pure act of reasoning without help or hindrance from bonds with the particular verbal structure and vocabulary of the problems. Whether or not they should be, they are not."

It might mean too that some of the work in problem solving might be put later in the course; for, if the object is to give the child knowledge of certain processes and relations, he evidently can get these much more easily at a later stage in his school life. There seems to be no good reason for trying to force on a grade V child something which takes a great deal of time and effort, and which may

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1 Thorndike - Psych. of Arith., p. 111

be impossible for him, if in the next year there has been developed an ability which will enable him to master it easily. At the same time there may be avoided a great deal of dissatisfaction and school hatred which comes from the continuous attempt to do something in which there is not the satisfaction which comes from success.

If this were done it would be necessary to substitute some other arithmetic activity. Oral problem work might be given greater and later prominence as a valuable means of increasing accuracy in the fundamental operations. This would also give an opportunity for development of the problem attitude at a level where there is comprehension, and perhaps more important it would afford a means of increasing arithmetic vocabulary upon which so much of the later work depends.

An implication is that the school is asking the child to do something too difficult for him in the way of comprehension, and as a result he is forced back on methods of looking for cues in problems, or working them by methods of routine which are far removed from the method desired in what has been termed the "problem-solving" attitude. It is evidenced here that the "problem-solving" attitude is not to any appreciable degree present because the child is not able to understand the problem, and without understanding it he cannot have the attitude toward it.

Criticisms:

Of an investigation of this kind many criticisms may be offered. When working with the operations of another's mind nothing tangible can be measured and only results of its operation can be recorded and these may easily be attributed to the wrong causes entirely.

In this particular case ability to analyze problems was tested by means of the sorting of slips. The steps included on these no doubt constitute the essential elements in the problems used, but as to whether or not the sorting of them is analagous to the mental process which is termed "analyzing the problem" cannot be definitely known. It can only be assumed that it is so, and, in the absence of better tools, this device was used. Other factors, such as reading ability, influence its efficiency, and in drawing conclusions these must be kept in mind.

As well as a reading difficulty there was, in the case of the diagnostic question work, difficulty in expression by writing - the compositional element. As far as possible the questions called for brief answers to minimize this difficulty, but it must be present.

The grade range was small, covering only three grades. An extension either way might produce interesting results.

The number of cases was not large but they were taken from three schools at different parts of the city, which would help to make the sample a random one. In dealing with the age differences, however, the cases became very small at either end and so are not at all reliable.

Suggestions:

The suggestions offered are in the way of further or other investigations which might be conducted.

1. It would be of interest to use the method of slip analysis over a wider grade range. Tendencies are shown in the case of IV, V and VI which might well be examined in higher and lower grades. It would be necessary to use a different set of problems and perhaps best to take a problem for each grade considered, giving it also in the grade below and that above to get a coherent sequence.

2. There was some evidence found to indicate that the statements which pupils offer in giving written answers to problems are not an aid to accuracy as they are expected to be. A series of questions might be given to be worked in the regular way with statements being put down as facts are dealt with or operations performed. These same questions might then be given to other subjects to be worked without the use of statements. Then with the same pupils, by way of check, the procedure could be reversed with a new set of problems. It might be determined in this way which produces the greater accuracy, that which presumes to make the child analyze the problem as he does it, or that which requires only that he perform the mechanical operations.

3. The ability which has been suggested as developing through the grades in the appreciation of problem

situations is no doubt closely dependent upon age or upon mental age. This dependence might be found by giving mental tests as well as tests in analysis to determine at what age analysis of a certain degree of difficulty might be expected.

4. Problems of a generalized nature gave evidence as to what extent ability in the comprehension of a problem situation apart from numbers was possible. Working with pupils individually is much more satisfactory with this kind of exercise. It is possible to give all the explanation necessary and overcome any language difficulties the subject may have. A series of problems of this type, ranging in difficulty from very easy to quite hard, might serve to determine just what degree of abstraction is possible at various ages and with various kinds of problem material.

5. There was a suggestion in the grade VI data of this investigation that girls at this age surpass boys in analytic ability. Means might be devised for determining if this be true.

6. If, as is evidenced here, the ability to solve problems, considered in terms of getting correct answers, is much beyond that to explain the processes of a problem, and this latter is dependent upon the development of a native ability, it would be of interest to take a number of cases and determine by a course of specially designed

training whether this ability could be brought much above the general level. It might be found that it could, but at the same time it is possible that the ability to solve more difficult problems would keep well in advance.

In conclusion, the writer wishes to make acknowledgment of the very helpful consideration received from the principals of the schools and teachers of the classes in which experimental work was done. The extent to which their cooperation has made this investigation possible is fully appreciated.